ADDRESS

Delivered by the President, Lieut.-Gen. J. F. Tennant, on presenting the Gold Medal to Professor G. H. Darwin.

The Council has this year awarded the Gold Medal of the Society to Professor G. H. Darwin "for his work on Tides and their influence on the Figures and Motions of the Heavenly Bodies," and I have now to lay before you the grounds on which this award has been made.

It is some fifteen years since Professor Darwin commenced his researches into the history of the Solar System, and the striking character of some of his results has already modified so fundamentally our ideas of the course of planetary evolution that it may be well to recur for a few moments to the state of our existing knowledge before these investigations were undertaken. And fortunately Professor Darwin has left on record * a general but explicit statement of the problem or problems which presented themselves to his mind at a time (1877) when he was almost deterred from attempting their solution by the profundity of the mathematical difficulties involved. Looking back with him to this time, we may well admire the determination which has so persistently overcome not only these difficulties, but that of incessant personal ill-health. The Society is glad to welcome as its Medallist one so worthy the name of Darwin as to have worked, under circumstances where most men's energies would have flagged, at the theory of Planetary Evolution.

"The Nebular Hypothesis," to quote the words of our Medallist in 1877, "seems to contain no explanation of the diversity which the telescope shows us is to be found in the inclinations of planets to their orbits," for as the planets "were originally parts of the great solar nebula, there appears to be no reason why the axes about which they rotated should not be nearly parallel to the axis of rotation of the Sun." The explanation of the large inclinations of planetary axes must, therefore, be sought in some feature of their history not considered in the Nebular Hypothesis as far as it was developed by Laplace. It is this second chapter in the hypothesis which Professor Darwin's work supplies, though others doubtless remain to be written.

* *Phil. Mag.*, 1877 March; *Observatory*, vol. i., p. 13.

His first tentative suggestion for the origin of large obliquities was as follows:—The contraction of a rotating planet, from the nebulous stage downwards, involves an increase in the angular rotation, and thus a greater equatorial bulging. Now, the phenomena of precession depend on the attraction of the Sun, and of satellites on the equatorial protuberance and on the rotation; and if these latter are both increased as above, the obliquity of the axis will increase. The obliquity is, in fact, for a given planetary system, dependent merely upon the density, and has increased as the planet has consolidated from a nebula into a compact mass. The mathematical expression of this fact is

$$\text{Log tan (obliquity)} = \text{A} - \frac{\text{B}}{\text{density}}.$$

where A and B are two constants depending on the particular planetary system.

In recent times, therefore, even long before the geological epoch, the obliquity of the Earth's equator to the ecliptic cannot have seriously changed, for its mean density has remained nearly constant. A possible reason for qualifying this statement had been already * examined by Professor Darwin, and found insufficient—viz., that geological changes, such as the elevation of continents, do not conform to the law of contraction by cooling above considered. In the deduction of the above law connecting obliquity and density, the surface of the contracting spheroid has been assumed to be one of equilibrium; but when the Earth has so far contracted as to reach the geological epoch, the surface begins to wrinkle into continents and oceans, mountains and valleys. Professor Darwin's first important paper on planetary history had been devoted to the investigation of the influence of such changes on the Earth's axis of rotation, and though the paper has many other points of interest, it cannot now be referred to further than to quote the principal conclusion in the author's own words: † "If a chain of mountains or a tableland, many degrees in width and as high as the Himalayas, were to be slowly elevated all round the equator, the effect would be *pro tanto* the same as an increase in the compression of the earth—that is to say, to increase the obliquity of the ecliptic. Nevertheless, the effect in that direction of even so gigantic a change would merely be to cause the Arctic circles to approach the tropics by a few inches. As, then, such an elevation is far in excess of any which geology teaches us can have ever taken place, we are compelled to believe that the obliquity of the ecliptic has remained sensibly constant throughout geological history," as it would if the surface had remained one of equilibrium.

We can thus infer from the above formula and our present

* *Phil. Trans.*, 1877, part i. † *Obs.*, vol. i., p. 15.

knowledge of the obliquity of the axis and density of the earth what was the obliquity when its density was half or a tenth of the present, or when the matter composing the earth was more diffused still. But there are two difficulties. Firstly, the above formula was deduced by arguments applicable to the precession of rigid bodies; and it yet remains to examine how far it will apply on reaching a stage in the past history when the planets were fluid or nebulous. Secondly, as we follow the history backwards, and the earth increases in size with diminution of density, there comes a stage when its surface reaches the Moon; and after this the two must be considered as one body. Before this the precessional effects will depend on the Sun and Moon conjointly, but after this on the Sun alone, and there is thus a discontinuity in the solution of the problem. But "in making a numerical application to the case of the Earth, it is found that the change in the obliquity must have proceeded so very slowly that even *when the earth was so diffuse as to fill the lunar orbit* the obliquity of the ecliptic must have had nearly its present amount." * Pushing back the argument beyond this point (which is characterised by the author as a somewhat wild speculation) it is found that when the Earth's diameter was a thousand times as great as it is at present, the obliquity was only a few minutes of arc.

Attention is called to the phrase printed above in italics because the conditions under which the Moon separated from the Earth were subsequently found by Professor Darwin to be so different; and I shall presently have to refer to this point as one of the most important results in his work.

Returning now to the history of changes in obliquity, and having disposed of the difficulty about the satellites of the other planets in a manner similar to that for the Moon and Earth, the question remains how far the formula for a rigid body is applicable to the history of non-rigid bodies. If it be applicable, then a study of the constants involved shows that the obliquities of the planets should decrease with distance from the Sun, which is not supported by observed facts; and it is thus probable that on taking account of the want of rigidity, fluidity, or nebulosity of a planetary mass we should find a different general result. The continuous alterations in form to which the yielding matter composing the planet would be subjected by the attraction of external bodies may be called bodily tides; and the problem may be regarded as the study of tides in the most general sense of the word. To this extension of our knowledge of planetary history beyond the limitation to the case of rigid bodies Professor Darwin has accordingly applied himself, with a success which the Council of the Society recognises in the present award, and which I will endeavour to describe to you.

The history of a planetary system may be traced in either

* *Obs.*, vol. i., p. 16.

direction, forwards or backwards in time. We may commence with a rotating and slowly contracting nebula, examine the conditions of its stability, and those under which a ring or satellite will detach itself when the stability breaks down, and follow the changes of the multiple system under the influence of gravitation. This is no doubt the ideal method of considering the question. But the generality of this problem, and the mathematical difficulties of dealing with fluids and gases, place it beyond the range of possible solution in practice. Professor Darwin has, however, recently considered * a particular case of the problem from this point of view—viz., that of a rotating fluid ellipsoid which becomes so elongated as to be unstable.

The other method of considering planetary history is to take an existing system, such as that of the Earth and the Moon, and trace its history backward. This has two great practical advantages. Firstly, although, as we have seen, perfect rigidity is inadmissible, the bodies can be treated as approximately rigid, which simplifies enormously the mathematical formulæ involved; and secondly, the general case can be at any time limited by the insertion of known numerical values for the arbitrary constants which present themselves. In this latter connection it is obviously advantageous to select as our particular case that of the Earth and Moon, where we have a far more complete knowledge of the numerical constants than in the case of other planets. Even in this instance, however, our knowledge is found to be lamentably deficient. Professor Darwin is often compelled to be content with a rough and even avowedly faulty assumption in the absence of numerical data, and accordingly some of his conclusions must be received with caution until these deficiencies can be supplied; and although his results have been occasionally quoted in an absolute form which he would be the first to disapprove, a reference to the original papers will always show clearly how far such general statements must be qualified.

If, in what follows, similar omissions of the necessary qualifications are occasionally found, the necessary brevity of this summary must be the excuse.

The work of our Medallist in solving the problem here indicated is to be found chiefly in five memoirs read to the Royal Society:

(1) "On the Bodily Tides of Viscous and Semi-elastic Spheroids, and on the Ocean Tides upon a yielding Nucleus." †

(2) "On the Precession of a Viscous Spheroid, and on the Remote History of the Earth." ‡

(3) "Problems connected with the Tides of a Viscous Spheroid." §

* *Phil. Trans.*, 1887, and *Proc. R. S.*, 1886.

† *Phil. Trans.*, 1879, pt. i.

‡ *Ibid.*, 1879, pt. ii.

§ *Ibid.*, 1879, pt. ii.

(4) "On the Secular Changes in the Elements of the Orbit of a Satellite revolving about a tidally distorted Planet." *

(5) "On the Tidal Friction of a Planet attended by several Satellites, and on the Evolution of the Solar System." †

The first of these papers, "A necessary first chapter to the investigation of the precession of imperfectly elastic spheroids," deals with the forced oscillations, first of a viscous, and ultimately of an elastico-viscous sphere. Sir W. Thomson had already treated the problem of a perfectly elastic sphere, and Professor Darwin shows how his solution may be extended to the case of a viscous sphere, and next of a sphere whose elasticity is not perfect but breaks down under continued stress, the forces requisite to maintain the body in any strained configuration diminishing in geometrical progression as the time increases in arithmetical progression. As an illustration of the necessity for caution in accepting the conclusions, I may quote Professor Darwin's words as to this assumption made at the very outset of the investigation. "There can be no doubt that all bodies *do* possess an imperfection in their elasticity of this general nature, but the exact law here assumed has not, as far as I am aware, any experimental justification; its adoption was rather due to mathematical necessities than to any other reason." If, then, we admit this assumption, it is shown how the problem in an imperfectly elastic sphere may be deduced from that in an elastic sphere. I need not further dwell upon the form of the solution, but a result obtained incidentally in the course of the work is of considerable practical interest. The author digresses from the main problem to consider the *ocean* tides on a nucleus such as we have considered. If there were two bodies precisely similar in form to the Earth, one of which was absolutely rigid except for the water on its surface, and the other composed of yielding matter, the apparent tides on these two bodies would differ, although the shape of the actual surface of the ocean on both might conceivably be the same, for the apparent tide in the first would be measured by the total displacement of the water, while in the second it would be the difference between the displacements of the water and the land. The failure of the observed ocean tides to agree with their theoretical values for a rigid nucleus will thus give an indication of the want of rigidity of the Earth's nucleus; and by a numerical interpretation of his equations Professor Darwin finds that unless the viscosity of the Earth were very much larger than that of pitch at near the freezing point, when it is hard, apparently solid and brittle, the viscous sphere would comport itself sensibly like a perfect fluid, and the ocean tides would be quite insignificant. It follows, therefore, that no very considerable

* *Phil. Trans.*, 1880, pt. ii. † *Ibid.*, 1881, pt. ii.

portion of the interior of the Earth can even distantly approach the fluid state.*

This conclusion as to the great rigidity of the Earth is confirmed several times in the course of Professor Darwin's work, and especially in a paper read to the Royal Society in 1881, "On the Stresses caused in the Interior of the earth by the Weight of Continents and Mountains,"† of which only the principal conclusion can here be quoted—that "if the Earth be solid throughout, then at a thousand miles from the surface the material must be strong as granite. If it be fluid or gaseous inside, and the crust a thousand miles thick, that crust must be stronger than granite, and if only two or three hundred miles in thickness, much stronger than granite." ‡ Again, in the second paper quoted above, to which reference will immediately be made, another confirmation is deduced from the theory of the tides. It had been pointed out some twenty years before by Adams and Delaunay that there was an outstanding secular acceleration of the Moon's mean motion of some $3''$ per century which could not be explained by ordinary lunar theory. It was suggested that the effects of *ocean* tidal friction might be responsible for this acceleration. The idea of friction of the ocean as held back by the Moon, against the solid Earth rotating inside it, was already familiar; as also the theoretical consequence that the reaction on the Moon would produce an apparent acceleration of the mean motion. It remained for Professor Darwin to point out that if the Earth were not very nearly rigid, an analogous tidal effect throughout its mass would produce a very much greater outstanding secular acceleration than that actually observed, whether the Earth be assumed viscous or very nearly perfectly elastic. Lastly, in the Memoir numbered (4) it is shown to be probable that, if we trace the Earth-Moon system backwards towards the point of separation, the original lunar orbit will make a considerable angle with the plane of rotation of the primæval planet, unless the Earth's viscosity be very large.

These four independent investigations thus point to the same conclusion—that the Earth must be enormously stiff; and are strongly confirmatory of Sir William Thomson's view that the Earth is solid throughout.

I pass now to the principal investigation of the second Memoir, in which a new light is thrown on the possibilities of planetary evolution. We have seen how, in 1877, Professor Darwin contemplated the separation of the Moon from the Earth at a time when the Earth was so diffuse as to nearly fill the present lunar orbit; and the subsequent history would have represented the contraction of the Earth and Moon to their present dimensions, their centres of gravity remaining at

* See also *B.A. Report*, 1883.

† *Phil. Trans.*, 1882, pt. i., p. 187. ‡ *Ibid.*, p. 230.

approximately the same mean distance. As I have already remarked, the notion of progressive change in this mean distance, due to the tides raised in one body by the other, was already familiar. It was reserved for our Medallist to show that when the term "*tide*" was extended to include yieldings of the whole mass of either body, the tidal action might be so great as to completely alter the above history. It is shown to be probable that the Moon was detached from the Earth when the latter had contracted to nearly its present dimensions, and that the present magnitude of the lunar orbit is a direct result of tidal interaction. By a very elegant geometrical illustration Professor Darwin has shown in two subsidiary papers* how the history of a system such as that of the Earth and Moon may be traced in its essential features without any reference to actual time. The notion of time is introduced when we assign a law or a numerical value to the viscosity of either planet; but if, without doing this, we recognise that the total energy of such a system must degrade, the chronological order of events becomes apparent, though the lapse of time between successive events is unknown. Initially the two bodies are rotating as a rigid body: the day and the month are the same; the Earth always turns the same face to the Moon and the Moon to the Earth; and they are nearly, if not quite, in juxtaposition. This configuration, however, being one of maximum energy, is essentially unstable, and the two bodies would gradually separate, the rotations of both being retarded, but that of the smaller much more rapidly than that of the larger. Under certain conditions, indeed, the diminution of rotation of the smaller might so far keep pace with its recession that the habit of always turning the same face to it might be sensibly retained throughout; and in the case of our Moon this habit was probably acquired very early in the history of the Earth. But with the parent body it would be different. The tides raised in it by the gradually receding satellite would indeed retard its rotation; but for some time the enlarging of the orbit of the satellite would increase its period of revolution much more rapidly, so that the number of days in a month (adopting the specific terms of our own system) would increase from the initial unity. But not indefinitely. After reaching a maximum, they would again diminish to unity, and we should ultimately reach a stage when the Earth and Moon were rotating as a rigid body, but at a considerable distance from each other. The tidal interaction of the two would be exhausted, and the configuration would be now one of minimum energy, and therefore stable. From this point their history would be concerned with the action of the Sun and other external bodies.

* "The Determination of the Secular Effects of Tidal Friction by a Graphical Method," *Proc. R. S.*, 1879; and "On the Analytical Expressions which gave the History of a Fluid Planet of Small Viscosity attended by a Single Satellite," *Proc. R. S.*, 1880.

In the geometrical construction above referred to Professor Darwin shows how these two limiting configurations are determined by the solution of the biquadratic equation

$$x^4 - hx^3 + 1 = 0,$$

which cannot have more than two real roots. In the simplest case of the problem, when the satellite is taken to be a point revolving in an orbit perpendicular to the axis of rotation of the planet, the equation appears at once; and in the more general case the expression on the left also occurs as the denominator in various integrals involved. The equation was originally obtained by seeking the condition that the Moon should always turn the same face to the Earth; but its fundamental significance is better represented by considering it as the condition that the energy of the system should be a maximum or minimum.

Now, in Memoir No. (2) of the five quoted above, Professor Darwin undertakes to trace the history of the Earth-Moon system backwards from its present configuration towards that of maximum energy on the hypothesis of a viscous Earth, and he obtains the surprising result that the internal tidal friction of such a viscous Earth would be sufficient to explain the present recession of the Moon, supposing it to have been separated from an Earth of nearly the present dimensions. He calculated the law of change of the day, the month, and the Moon's mean distance at the present time, and thus reduced previous concomitant values of the day, the month, and the Moon's mean distance. He finds that the number of days in a month, after increasing slightly for a time, diminishes as we go backwards (for one of the most interesting subsidiary results of the Memoir under consideration is that our Earth and Moon have at the present time passed through that configuration referred to above, when the number of days in a month is a maximum), and at the same time the distance of the Moon decreases. But the important point discovered by Professor Darwin is that when the day is as long as the month the Moon has nearly reached the surface of the Earth as we know it. The following table shows the course of the changes as we look backward from the present time :—

Time in millions of years.	Sidereal day in M.S. hours.	Moon's sidereal period in M.S. days.	No. of days in month.	Moon's distance in Earth's mean radii.
	h	d		
0·00	23·93	27·32	27·40	60·4
46·30	15·50	18·62	28·83	46·8
56·60	9·92	8·17	19·77	27·0
56·80	7·83	3·59	11·01	15·6
56·81	6·75	1·58	5·62	9·0
...	5·60	0·23	1·00	1·5

The whole of these results, excepting those in the last line, are derived from an integration by quadratures, on the hypothesis of a certain degree of viscosity of the Earth's mass. That degree of viscosity was so chosen as to be competent to produce the change of configuration in about the shortest possible times. Thus the time-scale in the first column must be regarded as a scale of inferior limits. It might be supposed from an inspection of this table that the changes took place at an ever-increasing rate as we look more and more remotely into the past, but this is not the case: for we finally come to a time when the changes proceeded slowly. The degree of retardation is, however, undetermined, and thus the length of the day when the month was equal to it can only be deduced from the equation of conservation of angular momentum, and this equation gives us no indication of the elapsed time.

"It is particularly important to notice that all the changes might have taken place in fifty-seven million years; and this is far within the time which physicists admit that the earth and moon may have existed. It is easy to suggest a great many *veræ causæ* for changes in the planetary system, but it is ingeneral correspondingly hard to show that they are competent to produce any marked effects without exorbitant demands on the efficiency of the causes and on lapse of time. It is a question of great interest to geologists to determine whether any part of these changes could have taken place during geological history," and on consideration this does not seem impossible.

Further, seeing that the results obtained point strongly to the conclusion that, if the Moon and Earth were ever molten viscous masses, then they once formed parts of a common mass, we are led to the inquiry as to how and why the planet broke up. "The conditions of stability of rotating masses of fluid are unfortunately unknown, and it is therefore impossible to do more than speculate on the subject. The most obvious explanation is similar to that given in Laplace's nebular hypothesis, namely, that the planet, being partly or wholly fluid, contracted, and thus rotated faster and faster until the ellipticity became so great that the equilibrium was unstable, and then an equatorial ring separated itself, and the ring finally conglomerated into a satellite. This theory, however, presents an important difference from the nebular hypothesis, in as far as that the ring was not left behind 240,000 miles away from the Earth when the planet was a rare gas, but that it was shed only 4,000 or 5,000 miles from the present surface of the Earth when the planet was perhaps partly solid and partly fluid. This view is to some extent confirmed by the ring of *Saturn*, which would thus be a satellite in the course of formation. It appears to me, however, that there is a good deal of difficulty in the acceptance of this view when it is considered along with the numerical results of the previous investigation." The chief difficulty is the smallness of the angular velocity indicated for the Earth at the time of rupture. "It seems

improbable that a rotation in a little over 5^h, with an ellipticity of one-twelfth" (very little in excess of that of *Jupiter*) "would render the system unstable."

Professor Darwin is thus led to a very ingenious suggestion as to the cause of rupture—viz., that the natural period of oscillation of the spheroid might be nearly coincident with that of the solar semi-diurnal tide, which would thus acquire a very large amplitude according to the elementary theory of vibrations. "Sir William Thomson has shown that a fluid spheroid of the same mean density as the Earth would perform a complete gravitational oscillation in $1^h\ 34^m$. The speed of oscillation varies as the square root of the density; hence it follows that a less dense spheroid would oscillate more slowly, and therefore a spheroid of the same mean density as the Earth, but consisting of a dense nucleus and a rarer surface, would probably oscillate in a longer time than $1^h\ 34^m$." *

It seems that tidal action may therefore have been responsible not only for much of the history of the Earth and Moon from their separation till the present time, but even for the separation itself, and it is hardly necessary to point out how great a modification of the Nebular Hypothesis this involves. I must again call your attention to the fact that large assumptions were made at the outset of these calculations. It was assumed that the Earth is homogeneous and viscous, and has a certain degree of viscosity, such that with the present length of day the semi-diurnal tide lags by $17°\ 30'$, which particular value makes the rate of change of obliquity nearly a maximum. Further, it was assumed that this viscosity remained constant as we trace the history of the Earth backward. Part of the calculations were repeated on a supposition of variable viscosity, and no serious modification of the results was found, which is so far satisfactory; and there are several instances in the investigations where the nature of the original hypothesis is found to have unexpectedly little influence on the results. Professor Darwin remarks that "whatever may be thought of the theory of the viscosity of the Earth and of the large speculations to which it has given rise, the fact remains that all the effects which have been attributed to the action of bodily tides would also follow, though probably at a somewhat less rapid rate, from the influence of ocean tides upon a rigid nucleus." This unimportance of the original hypothesis, indeed, almost seems to be characteristic of the whole subject, and no better illustration of it could be given than a conclusion of Sir William Thomson, several times quoted by Professor Darwin,† that the precession of a fluid spheroid would sensibly follow the same laws as though it were rigid. And we have already quoted the fact that the hypothesis of viscosity and nearly perfect elasti-

* *Phil. Trans.*, 1879, pt. ii., p. 537.

† See *Obs.*, vol. i., and Address to Section A, British Association, Glasgow, September 1876. The precession of a fluid spheroid has been ably treated by Mr. Bryan in a paper in the *Phil. Trans.* for 1889.

city of the earth would equally well explain the *amount* of the lunar secular acceleration, though in totally different ways.

In a later Memoir (No. (4) of the above series) Professor Darwin is led to modify his view of the initial state of the Earth-Moon system. In the table given above the effects of solar tidal friction have been practically neglected soon after leaving the present configuration. For as we go backwards the approach of the Moon to the Earth would produce a rapid increase of the lunar tides with reference to the solar. But as we approach the limiting configuration it is obvious that, though the tides raised in each body are large, the mutual tidal friction becomes small and ultimately vanishes when the two bodies rotate as a rigid body. The solar tidal friction can, therefore, not be neglected near this limiting configuration. On examination it is found that the effects of the solar tide would be slight save at the most remote period. On the whole, the effect of tidal friction would be to retard (looking backwards) the coincidence of the month and day, so that they would not reach equality until each was reduced to about 2 or $2\frac{1}{2}$ hours instead of $5^h\ 36^m$, and the surfaces of the two bodies would be nearly in contact, instead of there being even the small separation shown in the last line of the table. "Now it is a remarkable fact that the most rapid rate of revolution of a mass of fluid of the same mean density as the Earth, which is consistent with an ellipsoidal form of equilibrium, is $2^h\ 24^m$. Is this a mere coincidence, or does it not rather point to the break-up of the primæval planet into two masses in consequence of a too rapid rotation?"*

In this Memoir he also develops the general theory of the secular changes in the elements of the orbit of a satellite—that is to say, those elements which contain a description of the nature of the orbit; the mean distance, inclination, and eccentricity. It was remarked early in this Address that Professor Darwin's attention seems to have been originally attracted by the varied obliquities of the planetary axes to the ecliptic, and the whole of his work may be regarded as in a measure directed towards the quantitative explanation of these. In the paper on "Precession" he traced the changes in the obliquity of the Earth's axis sympathetic with those in the day, the month, and the distance of the Moon. But there the Moon's orbit was assumed to be circular and confined to the ecliptic. The introduction of eccentricity and inclination of the lunar orbit is found to modify the results of the former Memoir, and I have accordingly not referred to them. In the more complete discussion of Memoir (4) the changes in the inclinations (neglecting the eccentricity) of the lunar orbit are first followed backwards from the present configuration on two hypotheses—one that the viscosity is small, and the other that it is large. It has already been remarked that on the former supposition, when the day

* *Phil. Trans.*, 1880, pt. ii., p. 835.

and month are identical, we find the lunar orbit inclined at a considerable angle to the ecliptic; and in this case it would be difficult to believe that the Moon is a portion of the primæval planet detached by rapid rotation or by other causes. But by supposing the viscosity large enough, "we can trace the system back until the lunar orbit is sensibly coincident with the equator, and the equator is inclined to the ecliptic at an angle of 11° or 12°."* Next, the inclination of the lunar orbit is neglected for the study of changes in the eccentricity, and as an illustration of the interesting nature of such investigations I may quote one of the results obtained incidentally: †

"In the history of a satellite revolving about a planet of small viscosity, the circular orbit is dynamically unstable until 11 months of the satellite have become longer than 18 days of the planet. Since the day and month start from equality and end in equality, it follows that the eccentricity will rise to a maximum and ultimately diminish again."

Combining the two investigations, Professor Darwin shows how the history of the Earth-Moon system may be sketched from our present knowledge and the supposition that they originally formed part of the same planet.

"We begin with a planet not very much more than 8,000 miles in diameter, and probably partly solid, partly fluid, and partly gaseous. This planet is rotating in a period of from 2 to 4 hours about an axis inclined at about 11° or 12° to the normal to the ecliptic" (or even less if we include the effects of solar tidal friction; so that there is nothing extravagant in the supposition that it originally formed part of the Sun), "and is revolving about the Sun with a period not very much shorter than our present year." ‡ The planet separates into two masses, owing to rotational instability or tidal action, the larger being the Earth, and the smaller the Moon: they are nearly in contact with one another, and rotating nearly as though they were parts of one rigid body. The attraction of each distorts the other, and the Sun raises tides in both. In consequence of frictional resistance such a system is dynamically unstable. The Moon probably revolved a little slower than the Earth rotates—*i.e.*, the month was a little longer than the day—and this excess would tend to increase. Also "the axial rotation of the Moon is retarded by the attraction of the Earth on the tides raised in the Moon, and this retardation takes place at a far greater rate than the similar retardation of the Earth's rotation. As soon as the Moon rotates round her axis with twice the angular velocity with which she revolves in her orbit, the position of her axis of rotation (parallel with the Earth's axis) becomes dynamically unstable. The obliquity of the lunar equator to the plane of the orbit increases, attains a maximum, and then

* *Phil. Trans.*, 1880, pt. ii., p. 874.
† *Ibid.*, p. 877. ‡ *Ibid.*, p. 879.

diminishes. Meanwhile the lunar axial rotation is being reduced towards identity with the orbital motion.

"Finally her equator is nearly coincident with the plane of her orbit, and the attraction of the Earth on a tide which degenerates into a permanent ellipticity of the lunar equator causes her always to show the same face to the Earth."

As the month increased the lunar orbit became eccentric, and the eccentricity (never probably large) would reach a maximum and then continually decrease if the viscosity had remained unchanged; but as the Earth became more rigid and oceans with tides were formed, the decrease of eccentricity would gradually cease, and then again increase.

The plane of the lunar orbit is at first identical with that of the equator; but as the Moon recedes from the Earth the Sun's action is felt. Consequently the inclinations of the lunar orbit and equator to their proper planes increase to maximum values, and then decrease. Meanwhile these reference planes, which originally nearly coincided with the equator, open out, the inclination of the lunar proper plane to the ecliptic diminishing, while that of the terrestrial equator increases; and gradually the system arrives at its present state after an estimated period of fifty-four million years.

Professor Darwin thinks it hardly too much to say that if only sufficient time be granted for the process, and it be admitted that the diffused matter in the universe produces no material change in the motions of the Earth and Moon during the period of evolution, then some such system as ours must have been developed from a primæval planet, though the process might vary in detail. He says finally:—

"A theory, reposing on *veræ causæ*, which brings into quantitative correlation the lengths of the present day and month, the obliquity of the ecliptic, and the inclination and eccentricity of the lunar orbit, must, I think, have strong claims to acceptance."

Before passing from the study of the particular system which has most interest for us, and which, as we shall presently see, there is reason for believing to be unique in the solar system—that of the Earth and Moon—to the other planetary systems, attention may be briefly directed to the results of the third Memoir of the series, to which reference has not yet been made. In this several problems are discussed which suggest themselves in the course of the main investigation, but do not affect the main argument. The first of these is the distortion of the spheroid when it is subjected to tidal stress throughout its mass. "There is an unequal distribution of the tidal frictional couple in various latitudes. We may see in a general way that the tidal protuberance is principally equatorial, and that accordingly the Moon tends to retard the diurnal rotation of the equatorial portions of the sphere more rapidly than that of the polar regions. Hence the polar regions tend to outstrip the equator, and there is a

slow motion from west to east relatively to the equator." * But a numerical examination shows that no sensible effect of this kind can have occurred within recent geological times, although it may have been once sensible. It has had little or nothing to do with the observed crumpling of strata, but it may afford a possible explanation of the north and south trend of our great continents.

The second problem considered is that of the generation of heat by tidal friction. The kinetic energy lost by degradation must reappear as heat, which would have to be added to that due to the slow contraction of the Earth in any investigation as to its secular cooling. Numerical calculation, however, shows that the heat generated would be fairly represented by a temperature gradient from the surface downwards of about 1° Fahrenheit in 2,600 feet, whereas the observed temperature gradient, though very variable, is far greater: so that the effects of tidal friction are completely masked by those of contraction, and "Sir W. Thomson's investigation of the secular cooling of the Earth is not sensibly affected by these considerations." †

The third problem treats of the effects of inertia on the tides of a viscous spheroid, which have previously been shown to be unimportant for the purposes of the present inquiry.

In the fifth Memoir Professor Darwin passes from the Earth-Moon system to the other planets in detail. He has already made brief references to them in the fourth Memoir, pointing out how the small planet *Mars* fulfils our expectations that it should be far advanced in its evolution by presenting the only case in the whole system where a satellite is moving faster than the primary rotates, which will be the ultimate fate of our Moon; for "after the Moon's orbital motion has been reduced to identity with that of the Earth's rotation, solar tidal friction will further reduce the Earth's angular velocity, the tidal reaction on the Moon will be reversed and the Moon's orbital velocity will increase, and her distance from the Earth will diminish. But since the Moon's mass is very large, the Moon must recede to an enormous distance from the Earth before this reversal will take place. Now the satellites of Mars are very small, and therefore they need only to recede a short distance from the planet before the reversal of tidal reaction." ‡

When, however, the effects of tidal friction on the evolution of other planetary systems are considered numerically, Professor Darwin finds the startling result that the case of the Earth and Moon is probably unique. The separation of other satellites from their primaries, and of the planets themselves from the Sun, probably occurred in the manner suggested in the Nebular Hypothesis, and which was always assumed to be applicable to the Earth and Moon before the effects of tidal friction were investigated—

* *Phil. Trans.*, 1879, pt. ii., p. 588. † *Ibid.*, p. 593.
‡ *Phil. Trans.*, 1880, pt. ii., p. 883.

that is to say, the present orbits of the satellites roughly indicate the sizes of the parent bodies at the time of separation. Retrospectively, it is comparatively easy to recognise the singular character of our own system. It is a link between the planets which (as far as we know at present) are unattended and those which have satellites; and we have seen reason for believing that the separation of the Moon from the Earth was possibly more or less of an accident, and due to extraneous causes, such as the near coincidence of two harmonic periods. The abnormal character of our satellite is also indicated by its large size in comparison with the parent body; and when we proceed to the comparison of the orbital momentum of the satellites with the rotational momentum of the parent body, we find a ratio of 4·78 for our own system, while it seems improbable that this ratio can exceed ·04 for *Saturn*, the most favourable case among the other planets. On using such approximate numerical data as are available, Professor Darwin concludes that it appears unlikely that the satellites of "*Mars*, *Jupiter*, and *Saturn* originated very much nearer the present surfaces of the planets than we now observe them. But the data being insufficient, we cannot feel sure that the alteration of the dimensions of the orbits of these satellites has not been considerable. It remains, however, nearly certain that they cannot have first originated almost in contact with the present surfaces of the planets, in the same way as, in previous papers, has been shown to be probable with regard to the Moon and the Earth."

Further, the progressive decrease in the number of satellites as we pass from the outer planets to the inner may be connected with the increasing efficiency of solar tidal friction as we approach the Sun. The detaching of a satellite from a contracting planetary mass is probably a result of the increase of rotational velocity due to contraction. Now, solar tidal friction tends to check the rotation of the planet. For the outer planets the solar tidal friction would be small, and not sufficient to seriously retard the increase of rotation due to contraction, so that one or more satellites would be shed at successive epochs of instability. But for the nearer planets the influence of solar tidal friction would be greater; and for the Earth there may have been "for a long time nearly a balance between the retardation due to solar tidal friction and the acceleration due to contraction, and it was not until the planetary mass had contracted to nearly its present dimensions that an epoch of instability could occur." For the planets within the Earth, *Mercury* and *Venus*, not only are the effects of solar tidal friction manifested by the absence of satellites, but we have more recent evidence in the observations of M. Schiaparelli that its work is now complete. If M. Schiaparelli's conclusions are correct, the great solar tides have reduced these planets to the condition of our own Moon, rotating in the period of revolution round the primary.

In giving this brief sketch of the main outlines of Professor Darwin's work much has been omitted, for obvious reasons, that is of interest and importance. If some of the conclusions should appear speculative, it must be borne in mind that the main argument is the unexpected result of a mathematical investigation, and not an original speculation supported by subsequent analysis. For this reason the ideas recorded by Professor Darwin in 1877 have been contrasted with the results of his principal Memoir on the history of the Earth and Moon. There is no doubt that he has been at times hampered by the lack of sufficient data, and at others has not been able to represent known facts in our limited mathematical analysis; but if these are drawbacks from the point of view of great accuracy in the conclusions, from another standpoint they are advantages as indicating the deficiencies which it is important to supply in the future. Professor Darwin has shown how there may be correlations between the viscosity of the Earth, the change of obliquity of the ecliptic, and the Moon's secular variation; between the configuration of planetary systems and the rigidity of the matter composing them. And he has thus indicated how, in the future, one department of astronomy may benefit by increase of knowledge in another; and he has added a new link between astronomy and physics, which may in the future mean new opportunities for both. Let us hope that the time for taking advantage of them may not be delayed by any fault of astronomers!

Let us now turn for a few moments to the more practical work of our Medallist, which is also specified in the award. He has proved himself on several occasions skilful in experiments more or less related to his theoretical investigations. The reports which (in conjunction with his brother, Mr. Horace Darwin) he has made to the British Association on the lunar disturbance of gravity record a series of ingenious and very delicate experiments, which, unfortunately, disturbing causes rendered useless. And, although the subjects of our Medallist's theoretical work are generally beyond the reach of laboratory experiments, yet in such papers as that "On the Formation of a Ripple-mark in Sand" (a passing glance at the diagrams in which will show how ingeniously and prettily the complex nature of the liquid vortices is made manifest) and "On the Horizontal Thrust of a Mass of Sand" (whose value was acknowledged by the award of a medal by the Institute of Civil Engineers), we recognise a relationship to the study of the wrinkling of the Earth's surface into continents, and the stresses caused in its interior by their weight.

But Professor Darwin has done practical work of a most useful kind in another sense. It has in recent years fallen to his lot to undertake most of the laborious work necessary for the practical study of the ocean tides. Fifty years ago, though the general phenomena were known and some examination made of tides in various parts of the world, yet it was mainly in European seas

that extensive series of observations were available or that any attempt had been made to deal with them adequately; but the methods were imperfect, and the reductions required the constant exercise of much skill. The great advance of our modern knowledge is mainly due to the energy of the British Association and its committees. Two Reports, in 1872 and 1876, chiefly due to Sir W. Thomson, may be considered the first important step towards an extensive study by harmonic analysis. All the phenomena now, instead of being referred to the Sun's and Moon's hour angles, declinations, and parallaxes, were considered as harmonic functions of angles varying uniformly with the time; the entire phenomenon was looked on as the sum of a number of harmonic undulations depending directly on the Sun's and Moon's mean hour angles and longitude, &c., which increase directly as the time. This great simplification has enabled work to be undertaken which would previously have been impossible.

In the Report for 1883 Professor Darwin devoted himself to the revision and development of the method of these two reports and the production of a manual for harmonic analysis. "A committee," he remarks, "appointed for the examination of the question of the harmonic analysis of tidal observations practically finds itself engaged in the reduction of Indian tidal observations, since it is only in that country that any extensive system of observation, with systematic publication of results, exists." The Indian work was till comparatively recently carried out by Major (now Lieut.-Colonel) Baird, R.E., and has consisted in the observation and discussion of the tides at a number of places on the Indian coasts for which tide tables are now annually published by the Indian Government.

Since then something has been done elsewhere; but if we consider the great amount which might be done, the result is lamentably small. Forms for reduction have been printed, with full directions, but apparently the cost of reduction is still so great as to deter most Governments from undertaking the work on any large scale; and it may be admitted that in many cases the scientific interest is of greater importance than the practical value of discussing very large series of observations.

Recognising the importance of this consideration, Professor Darwin has recently turned his attention to supplying such data as will suffice for sailors, to whom it is of importance to know the times and heights of high and low water at certain points near the entrances to ports. Unfortunately, excepting in the North Atlantic, where the diurnal tides are very small, such information is very scanty and inaccurate. "When there is a large diurnal inequality, as is commonly the case in other seas, the heights and intervals, after the upper and lower lunar transits, are widely different; the two halves of each lunation differ much in their characters, and the season of the year has great influence. . . . The tidal information supplied by the Admiralty for such places consists of rough means of the rise

and interval at springs and neaps, modified by the important warning that the tide is affected by diurnal inequality." Such information being almost useless, Professor Darwin devised an approximate but very cheap and simple method (described in the Bakerian Lecture at the Royal Society about a year ago) for predicting tides at any port after a very moderate amount of computation. He is, it is believed, at present occupied with the further simplification of this method by some mechanical contrivances, and we may sincerely hope that his efforts may be successful and lead to a great practical improvement in tide tables everywhere.

If it were generally known that for any port whose tidal constants are known the curves could be furnished for a very small cost, I cannot but think that many ports where tidal observations have been made would go to the expense of having them discussed, and that tide gauges would be established to make observations.

The President, then delivering the Medal to Professor Darwin, addressed him in the following terms:—

Professor Darwin, I have now to complete my duty here by delivering to you the medal which the Council have awarded. As your father taught us how the action of admitted laws would allow us to trace back all living nature to the first slight germs of life and thus account for the wonderful and beautiful diversity around us, so you, in your turn, have endeavoured to trace back the fabric of the world and solar system to its remote origin. Neither your subject nor the mode of investigation you have necessarily followed will admit of your work meeting with that general assent which almost at once greeted your father's, but we hope that this medal may serve to assure you that its value is appreciated, and encourage you to perfect the outlines and fill up the details of the sketch you have given to us. That you may have health and strength to continue your labours is the best we can wish you.